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Optimal design of geometrically nonlinear structures using topology optimization

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ABSTRACT

This study deals with comparing the stiffness design of geometrically nonlinear structures and linear structures using topology optimization. Bi-directional Evolutionary Structures Optimization (BESO) is used for the design process. The geometrically nonlinear behavior of the structures is analyzed using a total Lagrangian finite element formulation and the equilibrium is achieved by Newton-Raphson iterative scheme. The topology optimization of linear and nonlinear modeling is performed. The sensitivity of the objective function is found with the adjoint method and the optimization problem is solved using BESO's update method. Objective function of complementary work is evaluated. A special technique, the continuation method, is applied to eliminate the instability of nonlinear structure optimization. ANSYS APDL is also used to do FEA of optimal topology to verify the effectiveness of geometrically nonlinear modelling. The results show that differences in stiffness of structures optimized using linear and nonlinear modelling are generally small but it can be large in some cases, especially structure highly involving buckling behaviour.

***Keywords**: BESO, linear and non-linear structure, topology optimization*

1. Introduction

Topology optimization became one popular research subject in structure a few decades ago. It finds not only structure design but also compliant mechanism design to meet the best performance. The requirement for light-weight, low-cost and so on put topology optimization in a high position. And, most of the work done is based on linearity behavior which is not always valid for applications involving large deformation. That is the motivation of this research.

Most of work done in topology optimization is based on linear behavior, assuming that the structures with linear elastic materials undergo small displacement. Linear structures can cover a large range of structural design problems. However, in many cases required nonlinear solutions because of large deformation which energy absorption structures and compliant mechanisms can be counted. And it can be called a geometrically nonlinear structure.

There have been several prior works that considered geometrical nonlinearity in topology optimization problems. It was introduced as a method for solving the topology optimization problems of geometrically nonlinear structures and compliant mechanisms (Brun & Tortorelli, 1998). The examples provided in the above-mentioned works were not able to clearly show a significant difference in the converged topologies or values of the objective function between linear and nonlinear modeling (Buhl, Pedersen & Sigmund, 1999). With the examples provided, it showed that in many cases, the solutions from the nonlinear modeling are only slightly different from the linear ones. However, if snap-through effects are involved in the problems, the difference could be significant. It proposed a microstructure-based design approach with a nonlinear FE formulation for the topology optimization of structures with geometrical non-linearity (Gea & Luo, 2001). It was considered topology optimization of non-linear compliant mechanisms represented with frame elements (Pedersen, Buhl & Sigmund, 2001). An element removal and reintroduction strategy for topology optimization problems with geometrical nonlinearity were proposed (Bruns & Tortorelli, 2003). A level set-based topology optimization method was developed for large deformation problems (Ha & Cho, 2008). BESO was applied for topology optimization of geometrically nonlinear structures under both force loading and displacement loading (Huang & Xie, 2008).

2. Geometrically Nonlinear Structures Optimization Method

All the methods for topology optimization used in this study are explained in detail here. Firstly, a typical optimization method called BESO is clarified. Secondly, it is very important to figure out the differences between geometrically linear analysis and geometrically nonlinear analysis. Based on that issue, the finite element method affecting the optimal topology using nonlinearity modelling is investigated (assuming that the structures undergo large displacement with small strain).

2.1. Bi-directional topology optimization method

The original BESO appears to search for the minimum material volume subject to given mean compliance or displacement. Topology optimization is often aimed at searching for the stiffest structure with a given volume of material. In BESO methods, a structure is optimized by removing and adding elements simultaneously. That is to say that, the element itself, rather than its associated physical or material parameters, is treated as the design variable (Huang & Xie, 2010). Thus, the optimization problem with the volume constraint is stated as

$$\begin{aligned} &\text{Minimize: } C = \frac{1}{2}F^T u \\ &\text{Subjec to: } V^* - \sum_{j=1}^{N} V_j x_j = 0 \\ &\qquad 0 \le x_{min} \le x_j \le 1 \end{aligned} \tag{2. 1}$$

where F and u are the applied load and displacement vectors and C is known as the mean compliance. V_j is the volume of an individual element and V* is the prescribed total structural volume N is the total number of elements in the analysis domain. The design variable x_j represents the density of individual elements limiting between prescribed x_{min} and 1. Notice that the material interpolation scheme (Bendsøe, 1989) has been applied.

2.2. Structural nonlinearity

2.2.1. Types of structural nonlinearity

Three main nonlinear behaviours are boundary nonlinearity, material nonlinearity and geometrical nonlinearity.

Boundary nonlinearity is caused by independent displacement on the external boundary condition. Contacts produce stresses and friction affected on changing in deformation.

The elastic material can generate nonlinearity behavior. The generated equation is expressed as $\sigma = E(\varepsilon)\varepsilon$, in which Young's Modulus is no more constant but proportional with strains. Elastoplastic, viscoelastic and viscoplastic can be included in nonlinear material behaviors.

In addition, geometrical nonlinearity is generated by nonlinear relationships in kinematic quantities like strains and displacement. Large displacement, large rotation and large strain are the roots of geometrical nonlinearity. In the linear model, displacements are small so that the effect of geometrical changing can be ignored. But, when that change becomes bigger, it has to be counted on the global stiffness matrix.

2.2.2. Incremental-iterative approach

Using the accumulated displacement, the resistant force (F) is obtained and the unbalanced force $({}^{t}R - {}^{t}F)$, which is the difference between the applied and the resistant forces, is determined. The iterative process at this load increment continues by calculating a new tangent stiffness matrix, finding the displacement and the unbalanced force (Figure). The equations used in the Newton-Raphson method can be stated as (Bathe, 2006).

$$\begin{aligned} &{}^{t+\Delta t}K_T^{(it-1)}\Delta u^{(it)} = {}^{t+\Delta t}R - {}^{t+\Delta t}F^{it-1} \\ &{}^{t+\Delta t}\Delta u^{(it)} = {}^{t+\Delta t}u^{it-1} + \Delta u^{(it)} \end{aligned} \tag{2. 2}$$

where Δt is a suitably chosen time increment and it denotes the iteration number of the Newton-Raphson procedure in each time increment. The initial conditions at the start of each time increment are:

$$^{t+\Delta t}u^{z(0)} = {}^{t}u; \quad {}^{t+\Delta t}K_T^{(0)} = {}^{t}K_T; \quad {}^{t+\Delta t}F^{(0)} = {}^{t}F \qquad (2.\ 2)$$

Convergence is achieved when both the errors, measured as the Euclidean norms of the unbalanced forces and the residual displacements, are less than a minimum value. The complete equilibrium path can be traced by finding the subsequent solution points at higher load levels using the same approach.

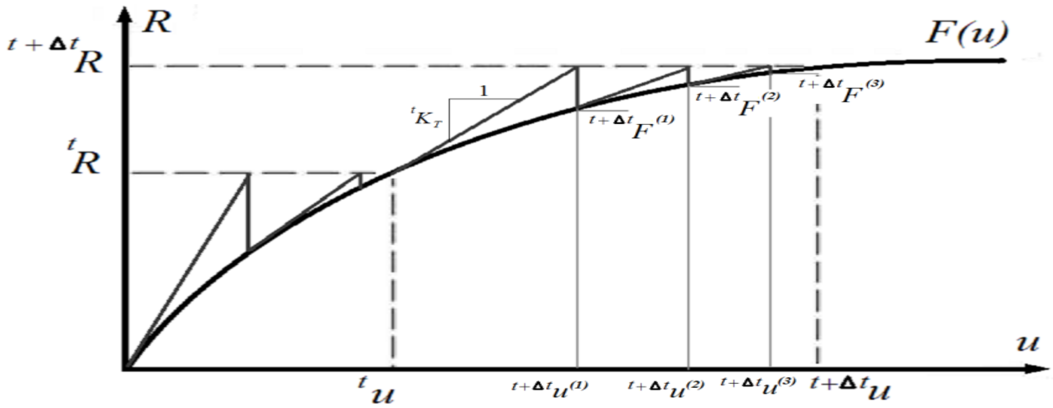


Figure 1. Illustration of incremental Newton-Raphson approach (Abdi, 2015).

3. Results and discussion

Two numerical examples are given to perform the optimization procedure. BESO is further developed to enable the topology optimization of geometrically nonlinear structures undergoing large deformation. This is archived using total Lagrangian FE formulation and an incremental iterative Newton-Raphson procedure to determine the equilibrium solution at every evolutionary iteration. For example, the continuation method and two-volume constraints are applied. The solutions for geometrically linear modelling and geometrically nonlinear modelling are compared, and the necessity of nonlinear modelling for the topology optimization of geometrically nonlinear structures is investigated. Final optimal topologies are analyzed by ANSYS APDL.

3.1 Cantilever beam

Not many structures, which need to be optimized for stiffness, undergo large displacements (Buhl, Pedersen & Sigmund, 1999). The first example considers designing the stiffness of a slender cantilever beam. The initial dimension of a cantilever beam is 1*0.25*0.1m in length, width and thickness. Figure 2 shows the design domain of this example. It is constrained at one end and free at the other, which is subjected to a concentrated load in the middle. The covering volume of the final topology equals 50 percent of the initial domain volume. Material is linear elastic with Young's modulus E=3 GPa and Poisson's ratio $\upsilon = 0.4$. Also, an updated penalty method is employed in the optimization process. BESO starts from the full design which is subdivided using a mesh of 80*20 four-node plane stress elements. The BESO's parameters are: evolution rate ER=2 percent; rmin=75mm; convergence criterion: 0.1 percent.

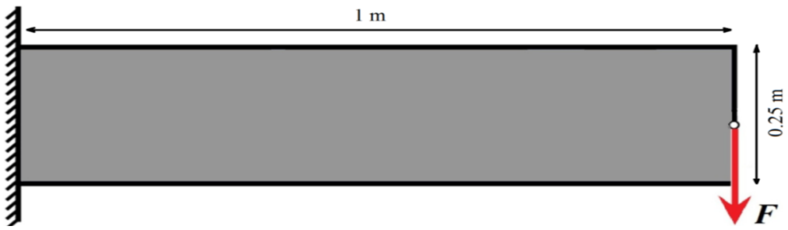


Figure 2. Design domain and boundary conditions of the geometrically nonlinear cantilever beam

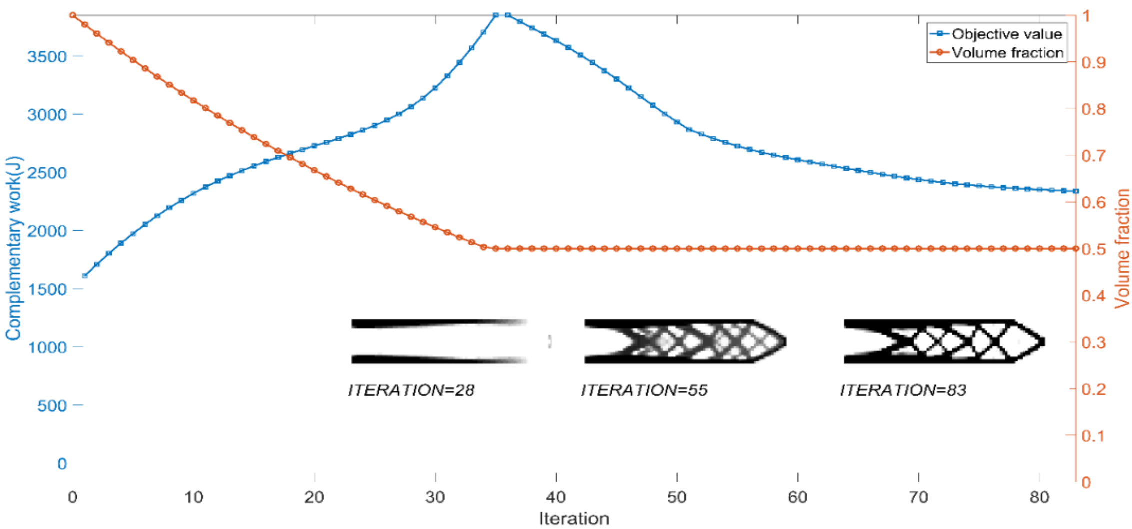


Figure 2. Evolutionary history of complementary work and volume fraction using BESO's update scheme and linear modelling subject to load case of 60 kN

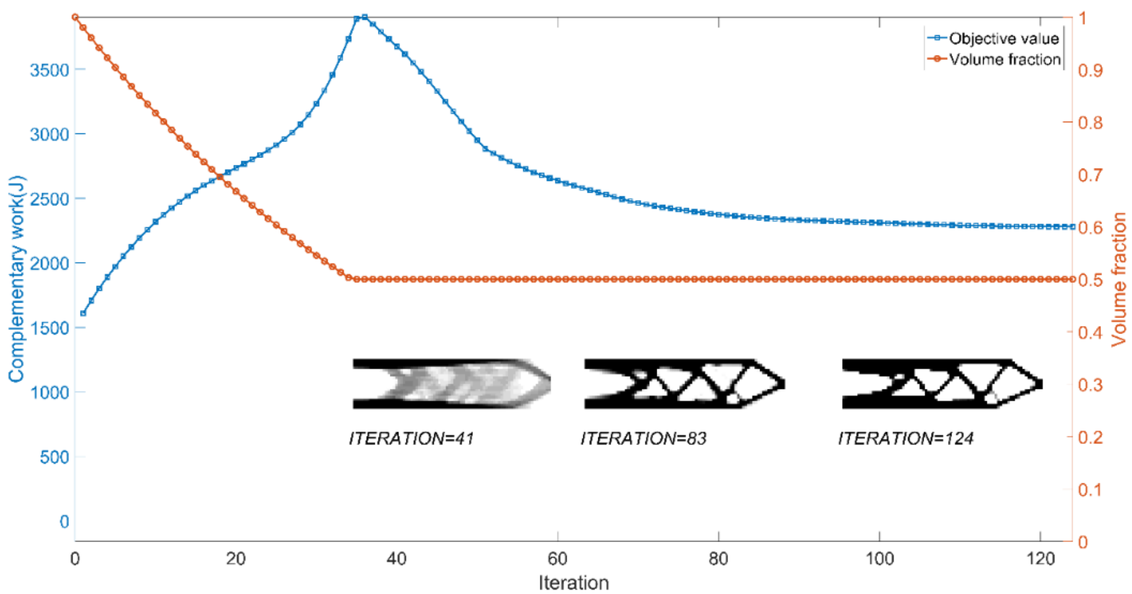


Figure 4. Evolutionary history of complementary work and volume fraction using BESO's update scheme and nonlinear modelling subject to load case of 60 kN

Several observations can be drawn from all these processes. In some initial iterations, because the effect of ER, elemental densities are not classified clearly with BESO. that effect can be seen in topology. It is noticed that the “optimal” topologies obtained for linear modelling are symmetric. With nonlinear modelling, topology is not symmetric.

The purpose of optimal design is that the displacement of topology is as small as possible. A comparison among four topologies is given in TABLE 1. Nonlinear displacement is the most important value which is supposed to minimize.

TABLE 1. Comparison of the optimization results of linear and nonlinear modelling using MMA and BESO at load case of 60 kN.

modelling	Update scheme	Objective value (J)	Linear FEA displacement (m)	Nonlinear FEA displacement (m)	Computation time (s)	iters
Linear	BESO	2327	-0.0778	-0.0776	25	83
Nonlinear	BESO	2279	-0.0763	-0.0758	2504	124

For all the topologies both the complementary work and displacement (linear and nonlinear) are listed. Generally, there is not much difference in this case. Nonlinear modelling can give a bit better result compared with linear. The biggest error is lower than 0.5 percent.

Because of the slightly lower complementary work of nonlinear modelling compared with linear, it could be argued that the difference is insufficient to justify the efficiency of nonlinear analysis. The next example, which can figure out the important role of nonlinear modelling essential, will be implemented.

3.2 Clamped beam

It can be seen that in the last example, the difference between linear and nonlinear modelling is less than 5 percent. In order to emphasize the differences between linear and nonlinear modelling, the example of the clamped beam which contains buckling behavior is given.

In addition, one more structural optimization model is considered. Figure 5. Design domain and boundary condition of clamped beam problem subject to centre load show a beam clamped at both ends. Concentration force is applied to the centre point of the top edge. The full topology is 8 m long, 1 m in width and 0.1 m in thickness. The final topology covers 20 percent of the volume. Material is linear elastic with Young’s modulus E=3 GPa and Poisson’s ratio $\upsilon = 0.4$. Here, the updated penalty method is not employed in the optimization process. Volume constrains is 20 percent. BESO starts from the design which is subdivided using a mesh of 320*40 four-node plane stress elements. The BESO parameters are: evolution rate ER=2 percent; rmin= 75 mm; convergence criterion 0.1 percent.

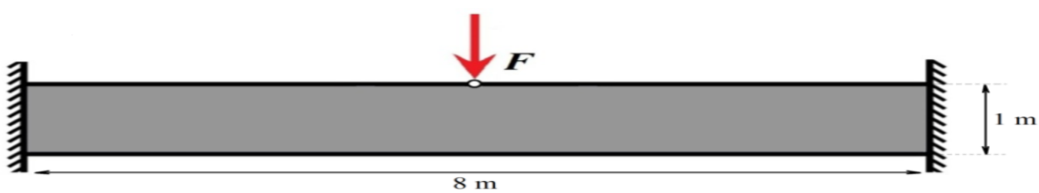


Figure 5. Design domain and boundary condition of clamped beam problem subject to centre load

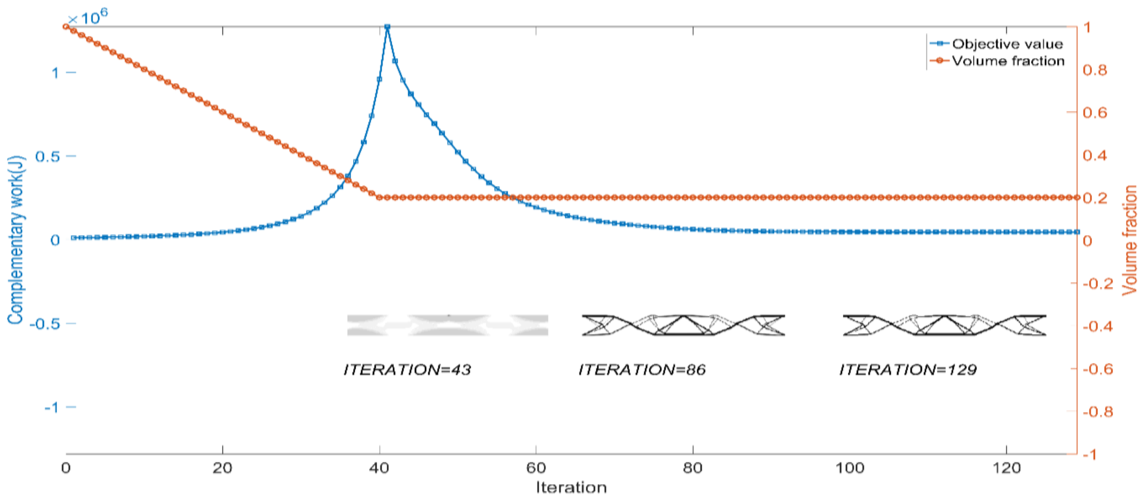


Figure 6. Evolutionary histories of complementary work and volume fraction using BESO's update scheme and linear modelling

The main difference between linear and nonlinear topology is two horizontal beams along the top edge of the domain. With a small load, it may be effective to resist the displacement. It is easy to collapse with a bigger load. Figure6 and **Error! Reference source not found.**7 show the evolutions of complementary work and volume fraction of topologies.

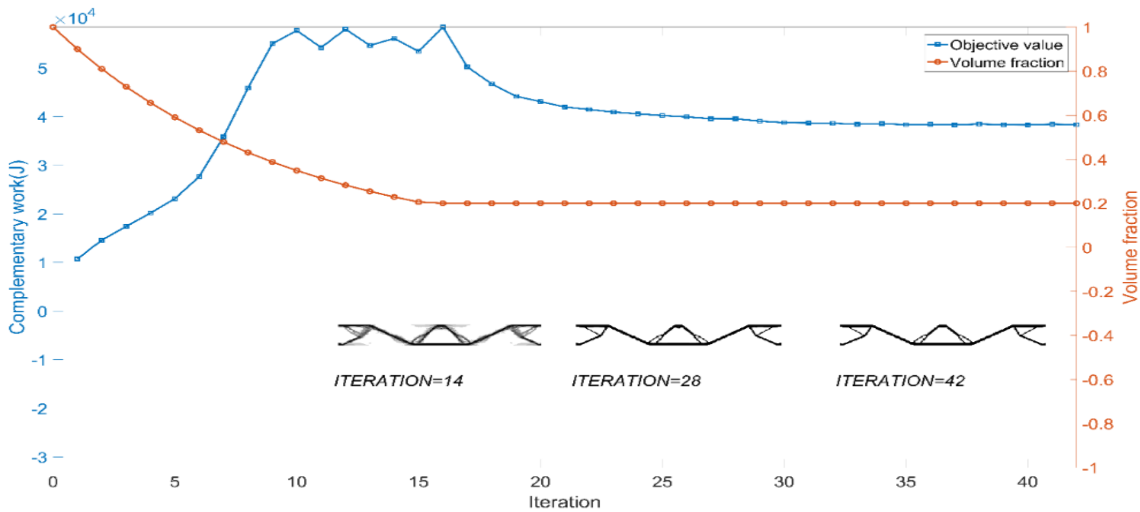


Figure 7. Evolutionary histories of complementary work and volume fraction using BESO's update scheme and nonlinear modelling

TABLE 2. Comparison of the optimization results of linear and nonlinear modelling using BESO

Modelling	Update scheme	Objective value (J)	Linear FEA displacement (m)	Nonlinear FEA displacement (m)	Computation time (s)	iters
Linear	BESO	45072	-0.2254	buckling	2576	129
Nonlinear	BESO	38328	-0.2156	-0.1925	23184	42

Not like the last example, totally different designs are obtained from nonlinear topology design comparing with linear topology design. Complementary work, which is the objective function, for nonlinear designs is much lower than linear designs. It means that the topology obtained using nonlinear modelling has a higher stiffness at the design load. Nonlinear results with BESO consist of a triangle in the middle of the structure under compression. With linear results, it also includes two horizontal bars on the upper edge.

The computational time required to obtain the nonlinear design is approximately seven hours. It is necessary in order to arrive at the optimal design. This cost of time is caused by a large number of elements and taking buckling behaviour into account.

A topology optimization procedure for the stiffness design of structures undergoing geometrically nonlinear deformations has been proposed. In many cases, computational time-consuming in geometrically nonlinear modelling is a big issue compared with the difference in the objective function. But in some case like a clamped beam, the difference in objective value may be large.

3.4 2D topology analysis using ANSYS APDL

Geometrically linear FEA and geometrically nonlinear FEA are implemented by using ANSYS APDL. Because geometrically nonlinear FEA is more complicated than a linear one, it is necessary to evaluate the reliability of the FEA solution using Matlab code. One of the most popular commercial software chosen is ANSYS APDL.

In order to evaluate solutions, all the character of the modelling in ANSYS APDL is set similarly to its modelling in Matlab code. Figure 8 shows the procedure of optimal topology FEA using ANSYS APDL.

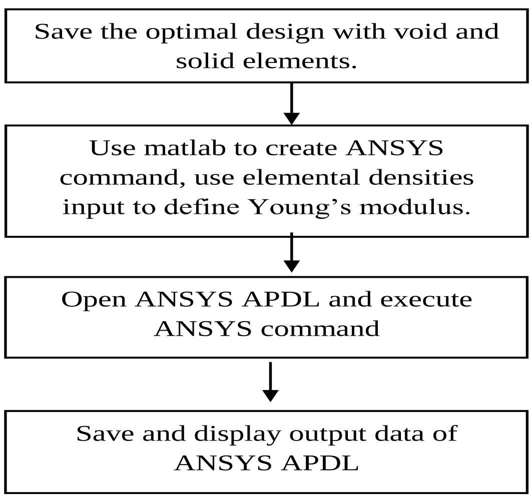
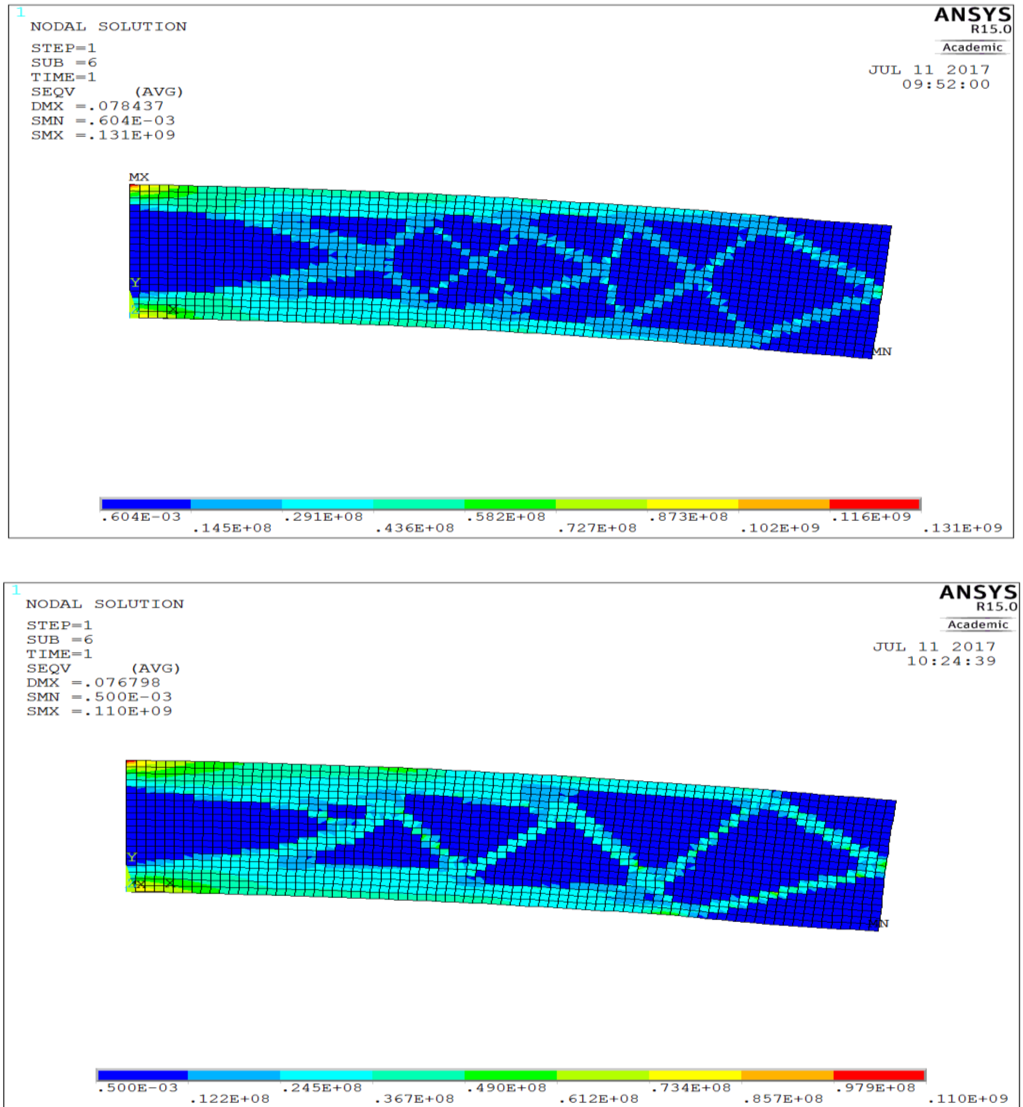


Figure 8. The procedure of optimal topology FEA using ANSYS APDL

The optimal topology of the cantilever beam is used as a test case to validate the developed ANSYS simulation. All the parameter is set similarly to the designed cantilever beam. The deflection of the beam is obtained from linear and nonlinear FEA using the developed ANSYS command and the results are compared with those obtained from analyzing the same problem using Matlab code. Figure .8 illustrates the optimal cantilever beam deflection using ANSYS FEA for both linear and nonlinear geometrical modelling.

TABLE 3. Comparison of the linear and nonlinear response of the optimal cantilever beam in Figure 9 using the Matlab FEA modelling and ANSYS FE modelling.

		Linear modelling	Nonlinear modelling
Linear FEA	Matlab FE modelling	-0.0778	-0.0763
	ANSYS FE modelling	-0.0778	-0.0763
Nonlinear FEA	Matlab FE modelling	-0.0776	-0.0758
	ANSYS FE modelling	-0.0777	-0.0759

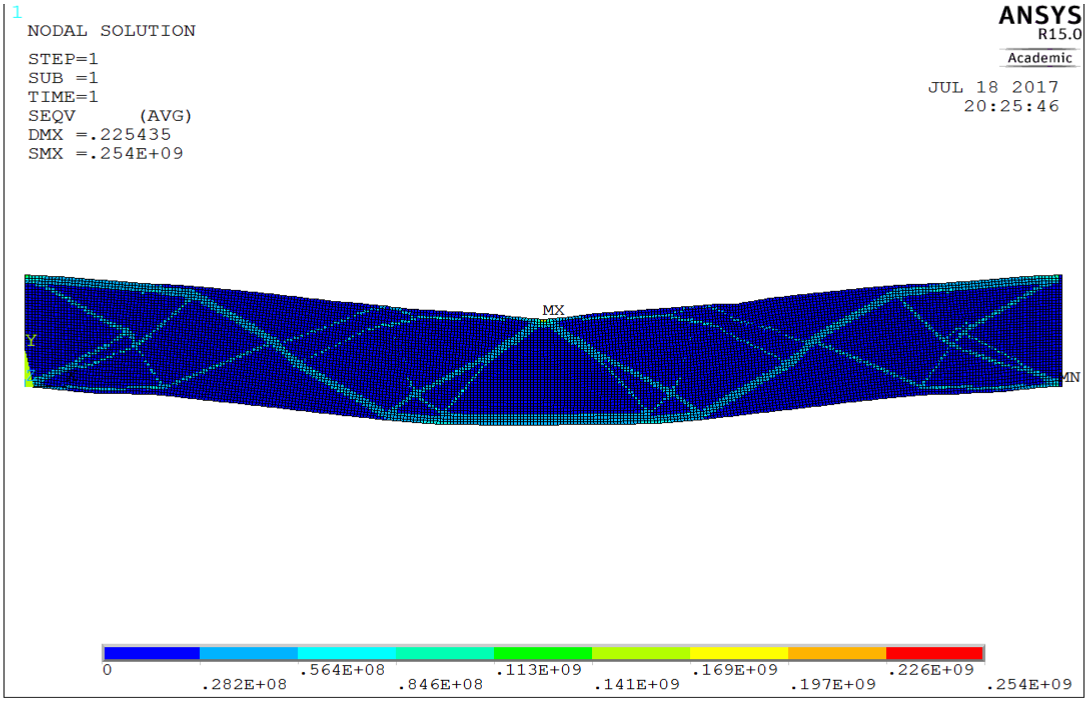


(a)

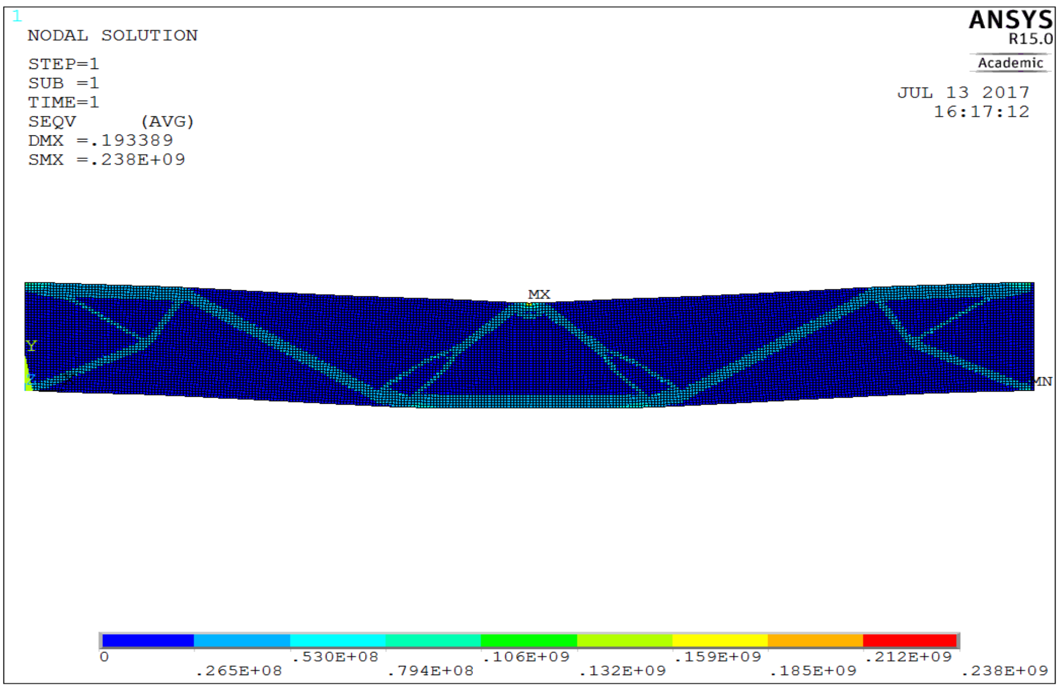
(b)

Figure 9. Illustration for the large deformation of cantilever beam subject to a designed load of 60 kN with (a) linear modelling with BESO's update method; (b) nonlinear modelling with BESO's update method

The is no difference in linear results. With nonlinear results, it can be seen the small gap. The error between FE modelling using ANSYS APDL and Matlab is lower than 1 percent.



(a)



(b)

Figure 10. Illustration for the large deformation of a clamped beam subject to a designed load of 400 kN with (a) linear modelling with BESO's update method; (b) nonlinear modelling with BESO's update method

TABLE 4. Comparison of the linear and nonlinear response of the optimal clamped beam in Figure obtained from the Matlab FE modelling and ANSYS FE modelling

		Linear modelling	Nonlinear modelling
		BESO	BESO
Linear FEA	Matlab FE modelling	-0.2254	-0.2156
	ANSYS FE modelling	-0.2254	-0.2156
Nonlinear FEA	Matlab FE modelling	buckling	-0.1925
	ANSYS FE modelling	buckling	-0.1934

Because of the results for optimal topology in Figure 9(a), there is no result for nonlinear FEA. With others, the result from ANSYS APDL is similar to Matlab. The error is less than 2 percent. Thus, the approach introduced is reliable for the FEA of optimal topology. There are many special behaviors of nonlinear modelling like locking, buckling and so on.

4. Conclusion

In those implemented example with topology optimization, the stiffness of structure is reduced slower than volume. The topology optimization results achieved from linear and nonlinear modelling showed that, for the presented examples, the solutions achieved from the optimization using non-linear modeling have a higher performance than those with linear modeling. Although there is not a significant difference between the solutions achieved from linear and nonlinear modeling in the first example of this study, nonlinear modeling consumes much more computational time than linear modeling. It is almost 50 times higher. The displacement of topologies using nonlinear modeling is lower than 3 percent compared with linear modeling. The results from the second example case which involves buckling (snap-through) effects, showed the importance of implementing nonlinear modeling in large displacement problems. The topology using linear modeling goes buckling when the applied force is lower 17 percent of the design load. And, the topology using nonlinear modeling goes buckling when the applied force is higher 40 percent than the design load.

Nonlinear FEA using ANSYS APDL is necessary to evaluate optimal topology. And, it is possible to use ANSYS APDL to build a finite element modeling. the difference between ANSYS FEA results and Matlab FEA results is less than 1 percent.

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